TEMPERATURE FIELD IN A PLATE CONTAINING A SYSTEM OF HEAT SOURCES

Yu. M. Kolyano, M. M. Semerak, and I. P. Dmitrash

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The temperature field is determined in a circular plate with a system of thin extrinsic heat sources.

In designing and preparing engineering structures and instruments, in particular electrovacuum instruments, it is of particular interest to determine and investigate the temperature field in plate-like elements with a system of extrinsic inclusions.

We will examine a circular plate with radius R, containing thin extinsic circular heat sources, which are positioned on concentric circles with radius $b_i (i = 1, 2, \ldots, m)$ (Fig. 1). The number of inclusions on the i-th circle is assumed to be n_i and their radius R_i, while the specific intensity of the heat sources $q_i(\tau)$ in the inclusions is assumed to vary with time. On each of the circles, the inclusions can be positioned as follows: Either the polar angle between the first and the n_i-th inclusions is $2\varphi_i$, and they are symmetrically placed relative to the radius of the plate, which we assume to be the origin for measuring the polar angle, and the rest of the inclusions are positioned between the ones indicated at equal distances, or one of them is placed on the radius, which serves as the origin for measuring the polar angle, and the rest of the inclusions are positioned cyclically along the circle. Convective heat exchange with the external medium with temperature $i_m(\tau)$ occurs through the lateral surfaces of the plate $z = \pm\delta$. The end-face surface of the plate is assumed to be thermally insulated.

In order to determine the nonstationary temperature field in the plate being examined with the inclusions we have the equation of heat conduction [1]

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\lambda(r,\varphi)\frac{\partial T}{\partial r}\right]+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[\lambda(r,\varphi)\frac{\partial T}{\partial\varphi}\right]-\frac{\alpha(r,\varphi)}{\delta}[T-t_m(\tau)]+W(r,\varphi,\tau)=C(r,\varphi)\frac{\partial T}{\partial\tau} \tag{1}$$

and boundary conditions

$$T|_{\tau=0}=0; \tag{2}$$

$$T|_{r=0}\neq\infty;\quad \left.\frac{\partial T}{\partial r}\right|_{r=R}=0; \tag{3}$$

$$\left.\frac{\partial T}{\partial\varphi}\right|_{\varphi=0}=0;\quad \left.\frac{\partial T}{\partial\varphi}\right|_{\varphi=\psi}=0. \tag{4}$$

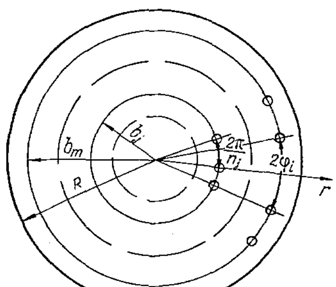

Fig. 1. Statement of the problem.

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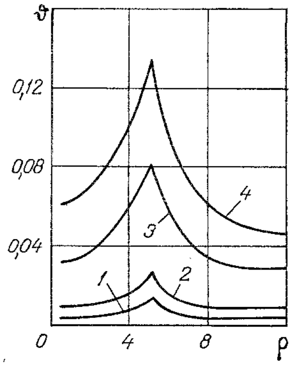

Fig. 2

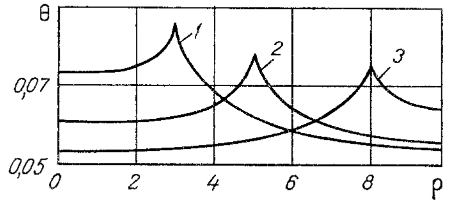

Fig. 3

Fig. 2. Computed temperature profiles for short times ($\varphi = 0$; $b_1^* = 5$): 1) Fo = 0.5; 2) 1.0; 3) 3.0; 4) 5.0.

Fig. 3. Temperature profiles for different radii for the circles on which the inclusions were placed, with $n = 6$; $\varphi = 0$: 1) $b_1^* = 3.0$; 2) 5.0; 3) 8.0.

Conditions (4) follow from the physical symmetry of the system. In the case being examined, it is clear that $\psi = \pi$, but in some particular cases the value of ψ can be different.

Let us replace the circular inclusion by an inclusion with a cross section shaped like a curvilinear trapezium, and in addition, the transverse cross-sectional areas in both cases must be identical. We will write down the thermophysical characteristics of such a system with the help of the characteristic functions [2] in the following form:

$$\beta(r, \varphi) = \beta_0 + \sum_{i=1}^{m} (\beta_i - \beta_0) M_i(r) \sum_{k=1}^{n_i} N_{ik}(\varphi), \tag{5}$$

where

$$M_i(r) = S(r - b_i + R_i) - S(r - b_i - R_i); \tag{6}$$

$$N_{ik}(\varphi) = S(\varphi - \varphi_{ik} + \varepsilon_i) - S(\varphi - \varphi_{ik} - \varepsilon_i); \tag{7}$$

$\varepsilon_i = \pi R_i/4b_i$ is chosen from the identity of the cross sectional surface areas; $\varphi_{ik} = [(2\pi - 2\varphi_i)/n_i^*](k-1) + \varphi_i$; $n_i^* = n_i - 1$ under the condition that $\varphi_i \neq 0$, and $n_i^* = n_i$, $\varphi_i = 0$ with cyclical positioning of inclusions along the circle;

$$S(x) = \begin{cases} 1, & x > 0, \\ \dfrac{1}{2}, & x = 0, \\ 0, & x < 0 \end{cases}$$

is the unit symmetrical function; the plate characteristics are labeled with the index zero, while i labels the inclusion on the i-th circle.

We will write the specific intensity of the heat sources in the form

$$W(r, \varphi, \tau) = \sum_{i=1}^{m} q_i(\tau) M_i(r) \sum_{k=1}^{n_i} N_{ik}(\varphi). \tag{8}$$

Taking into account the fact that the inclusions are thin, we will represent the characteristic functions in expressions (6) and (7) in the form [3]

$$M_i(r) N_{ik}(\varphi) = \frac{\sigma_i}{b_i} \delta(r - b_i, R_i) \delta(\varphi - \varphi_{ik}, \varepsilon_i), \tag{9}$$

where $\delta(x, h) = (S(x + h) - S(x - h))/2h$ with $h \to 0$; $\sigma_i = \pi R_i^2$ is the transverse cross-sectional area of an inclusion.

Substituting the thermophysical characteristics (5) and the specific intensity of the heat sources in system (8) taking into account the equality (9) in Eq. (1), applying the Laplace transformation with respect to time taking into account the initial condition (2) and the finite Fourier cosine transformation with respect to the angular variable in the limits from zero to ψ and the boundary conditions (4) and taking the limit for $R_i \to 0$, for which σ_i remains constant, we obtain the following equation for the transforms with impulse-type singularities on the right side:

$$\frac{d^2\bar{\tilde{T}}}{dr^2} + \frac{1}{r}\frac{d\bar{\tilde{T}}}{dr} - \left(\frac{\nu^2}{r^2} + \gamma^2\right)\bar{\tilde{T}} = f(r,\ p,\ s), \tag{10}$$

where

$$f(r,\ 0,\ s) = -\varkappa_0^2\psi\bar{\tilde{t}}_m + \sum_{i=1}^{m} A_i\delta(r - b_i) \quad \text{for} \quad p = 0;$$

$$f(r,\ p,\ s) = \sum_{i=1}^{m} B_i\delta(r - b_i) \quad \text{for} \quad p \geqslant 1;$$

$$A_i = \sum_{k=1}^{L_i} E_{ik}\Phi_{ik}(0); \quad \varkappa_j^2 = \frac{\alpha_j}{\lambda_j\delta}, \quad j = 0,\ 1,\ \ldots,\ m; \quad \nu = \frac{p\pi}{\psi};$$

$$B_i = \sum_{k=1}^{L_i}\left[E_{ik}\Phi_{ik}(p) - d_i\left.\frac{\partial\tilde{T}}{\partial\varphi}\right|_{\substack{r=b_i\\ \varphi=\varphi_{ik}}}\Omega_{ik}(p)\right];$$

$$E_{ik} = \omega_i\tilde{T}|_{\substack{r=b_i\\ \varphi=\varphi_{ik}}} - g_i\left.\frac{\partial\tilde{T}}{\partial r}\right|_{\substack{r=b_i\\ \varphi=\varphi_{ik}}} - \chi_i\tilde{t}_c - \tilde{Q}_i;$$

$$\omega_i = \frac{s}{a_0}(C_i^* - 1)\frac{\sigma_i}{b_i} + \chi_i; \quad C_i^* = \frac{C_i}{C_0}; \quad a_0 = \frac{\lambda_0}{C_0};$$

$$g_i = (\lambda_i^* - 1)\frac{\sigma_i}{b_i^2}; \quad \lambda_i^* = \frac{\lambda_i}{\lambda_0}; \quad \chi_i = (\varkappa_i^2\lambda_i^* - \varkappa_0^2)\frac{\sigma_i}{b_i};$$

$$d_i = \frac{g_i}{b_i}; \quad \tilde{Q}_i = \frac{\tilde{q}_i}{\lambda_0}\frac{\sigma_i}{b_i}; \quad \gamma = \sqrt{\frac{s}{a_0} + \varkappa_0^2};$$

$$\Phi_{ik}(p) = \int_0^{\psi}\delta(\varphi - \varphi_{ik})\cos\nu\varphi d\varphi, \quad p = 0,\ 1,\ 2,\ \ldots;$$

$$\Omega_{ik}(p) = \int_0^{\psi}\frac{d}{d\varphi}[\delta(\varphi - \varphi_{ik})]\cos\nu\varphi d\varphi, \quad p = 1,\ 2,\ 3,\ \ldots;$$

L_i takes values so that $\varphi_{iL_i} \leq \psi$, while $\varphi_{i(L_i+1)} > \psi$.

Finding the solution [4] of Eq. (10) taking into account the boundary conditions (3) in the case that $p = 0$ and $p \geq 1$, respectively, and applying the transformation formula for the finite Fourier cosine transform, we obtain the following expression for the Laplace transform of the temperature field:

$$\tilde{T}(r,\ \varphi,\ s) = \varkappa_0^2\frac{\tilde{t}_m}{\gamma^2} - \frac{1}{\psi}\sum_{i=1}^{m} b_i\left[A_iF_0(\gamma,\ R,\ b_i,\ r) + 2\sum_{p=1}^{\infty} B_iF_\nu(\gamma,\ R,\ b_i,\ r)\cos\nu\varphi\right], \tag{11}$$

where

$$F_\nu(\gamma,\ R,\ b_i,\ r) = K_\nu(\gamma b_i)I_\nu(\gamma r)S(b_i - r) + I_\nu(\gamma b_i)K_\nu(\gamma r)S(r - b_i) - P_\nu I_\nu(\gamma b_i)I_\nu(\gamma r);$$

$$P_\nu = \frac{\nu K_\nu(\gamma R) - \gamma RK_{\nu+1}(\gamma R)}{\nu I_\nu(\gamma R) + \gamma RI_{\nu+1}(\gamma R)}.$$

In order to determine the unknown quantities $\tilde{T}|_{\substack{r=b_i\\ \varphi=\varphi_{ik}}}$, $\left.\frac{\partial\tilde{T}}{\partial r}\right|_{\substack{r=b_i\\ \varphi=\varphi_{ik}}}$, $\left.\frac{\partial\tilde{T}}{\partial\varphi}\right|_{\substack{r=b_i\\ \varphi=\varphi_{ik}}}$, a system of $3\sum_{i=1}^{m} iL_i$ algebraic equations follows from (11):

$$\left.\begin{aligned}
&\tilde{T}|_{\substack{r=b_j\\ \varphi=\varphi_{jl}}} = \varkappa_0^2 \frac{\tilde{t}_{\mathrm{m}}}{\gamma^2} - \frac{1}{\psi} \sum_{i=1}^{m} b_i \Big[A_i F_0(\gamma, R, b_i, b_j) + \\
&\qquad + 2 \sum_{p=1}^{\infty} B_i F_\nu(\gamma, R, b_i, b_j) \cos \nu\varphi_{jl} \Big], \\
&\frac{\partial \tilde{T}}{\partial r}\Big|_{\substack{r=b_j\\ \varphi=\varphi_{jl}}} = -\frac{1}{\psi} \sum_{i=1}^{m} b_i \Big[A_i H_0(\gamma, R, b_i, b_j) + \\
&+ 2 \sum_{p=1}^{\infty} B_i H_\nu(\gamma, R, b_i, b_j) \cos \nu\varphi_{jl} \Big] \quad (l = 1, 2, \ldots, L_j, \quad j = \\
&\qquad = 1, 2, \ldots, m), \\
&\frac{\partial \tilde{T}}{\partial \varphi}\Big|_{\substack{r=b_j\\ \varphi=\varphi_{jl}}} = \frac{2}{\psi} \sum_{i=1}^{m} b_i \sum_{p=1}^{\infty} B_i \nu F_\nu(\gamma, R, b_i, b_j) \sin \nu\varphi_{jl},
\end{aligned}\right\} \tag{12}$$

where $H_\nu(\gamma, R, b_i, b_j) = \dfrac{dF_\nu(\gamma, R, b_i, r)}{dr}\Big|_{r=b_j}$.

Let us examine a plate with n extrinsic heat sources, cyclically placed on a single circle. In this case, in (11)

$$m = 1; \quad \varphi_1 = 0; \; n_1^* = n; \quad \varphi_{1k} = \frac{2\pi}{n}(k-1), \quad k = 1, 2, \ldots, n;$$

$$\psi = \frac{\pi}{n}; \quad L_1 = 1; \quad \nu = np.$$

Then

$$\tilde{T}(r, \varphi, s) = \varkappa_0^2 \frac{\tilde{t}_{\mathrm{m}}}{\gamma^2} - \frac{n}{\pi} A b_1 \Big[F_0(\gamma, R, b_1, r) + 2 \sum_{p=1}^{\infty} F_\nu(\gamma, R, b_1, r) \cos pn\varphi \Big], \tag{13}$$

where

$$A = 0{,}5 \Big(\omega_1 \tilde{T}|_{\substack{r=b_1\\ \varphi=0}} - g_1 \frac{\partial \tilde{T}}{\partial r}\Big|_{\substack{r=b_1\\ \varphi=0}} - \chi_1 \tilde{t}_{\mathrm{m}} - \tilde{Q}_1\Big).$$

In order to determine the quantities $\tilde{T}|_{\substack{r=b_1\\ \varphi=0}}$ and $\frac{\partial \tilde{T}}{\partial r}\Big|_{\substack{r=b_1\\ \varphi=0}}$ from (12) we obtain the following system of algebraic equations

$$\left.\begin{aligned}
G_{11} \tilde{T}|_{\substack{r=b_1\\ \varphi=0}} + G_{12} \frac{\partial \tilde{T}}{\partial r}\Big|_{\substack{r=b_1\\ \varphi=0}} = f_1, \\
G_{21} \tilde{T}|_{\substack{r=b_1\\ \varphi=0}} + G_{22} \frac{\partial \tilde{T}}{\partial r}\Big|_{\substack{r=b_1\\ \varphi=0}} = f_2,
\end{aligned}\right\} \tag{14}$$

where

$$G_{11} = 1 + \frac{n}{2\pi} \omega_1 b_1 D_1(\gamma, R, b_1); \quad G_{12} = -\frac{n}{2\pi} g_1 b_1 D_1(\gamma, R, b_1);$$

$$G_{21} = \frac{n}{2\pi} \omega_1 b_1 D_2(\gamma, R, b_1); \quad G_{22} = 1 - \frac{n}{2\pi} g_1 b_1 D_2(\gamma, R, b_1);$$

$$f_1 = \varkappa_0^2 \frac{\tilde{t}_{\mathrm{m}}}{\gamma^2} + \frac{n}{2\pi} b_1 (\chi_1 \tilde{t}_{\mathrm{m}} + \tilde{Q}_1) D_1(\gamma, R, b_1); \quad f_2 = \frac{n}{2\pi} b_1 (\chi_1 \tilde{t}_{\mathrm{m}} + \tilde{Q}_1) D_2(\gamma, R, b_1);$$

$$D_1(\gamma, R, b_1) = F_0(\gamma, R, b_1, b_1) + 2\sum_{p=1}^{\infty} F_\nu(\gamma, R, b_1, b_1);$$

$$D_2(\gamma, R, b_1) = H_0(\gamma, R, b_1, b_1) + 2\sum_{p=1}^{\infty} H_\nu(\gamma, R, b_1, b_1).$$

Determining the quantities $\tilde{T}|_{\substack{r=b_1\\ \varphi=0}}$ and $\left.\frac{\partial \tilde{T}}{\partial r}\right|_{\substack{r=b_1\\ \varphi=0}}$ from (14) and substituting them into expression (13), we have

$$\tilde{T}(r, \varphi, s) = \varkappa_0^2 \frac{\tilde{t}_m}{\gamma^2} - \frac{n}{2\pi} b_1 \frac{\left(\omega_1 \frac{\varkappa_0^2}{\gamma^2} - \chi_1\right)\tilde{t}_m - \tilde{Q}_1}{\Delta}\left[F_0(\gamma, R, b_1, r) + 2\sum_{p=1}^{\infty} F_\nu(\gamma, R, b_1, r)\cos p n\varphi\right], \tag{15}$$

where $\Delta = 1 + (n/2\pi)\omega_1 b_1 D_1(\gamma, R, b_1) - (n/2\pi) g_1 b_1 D_2(\gamma, R, b_1)$.

Assuming that the temperature of the external medium and the specific intensity of the heat sources in the inclusions vary in time according to the law

$$t_m(\tau) = t_0 S_+(\tau) \text{ and } q_1(\tau) = q_0 S_+(\tau), \tag{16}$$

we obtain

$$\tilde{T}(r, \varphi, s) = \varkappa_0^2 \frac{t_0}{s\gamma^2} - \frac{n}{2\pi} b_1 \frac{\left(\omega_1 \frac{\varkappa_0^2}{\gamma^2} - \chi_1\right) t_0 - Q_0}{s\Delta}\left[F_0(\gamma, R, b_1, r) + 2\sum_{p=1}^{\infty} F_\nu(\gamma, R, b_1, r)\cos p n\varphi\right], \tag{17}$$

where $Q_0 = \frac{q_0}{\lambda_0}\frac{\sigma_1}{b_1}$; $S_+(\tau) = \begin{cases}1, & \tau > 0,\\ 0, & \tau \leqslant 0\end{cases}$ is the asymmetric unit function.

From expression (17), we obtain an expression for the temperature field for an infinite plate in the absence of heat sources in the inclusions for short times in the section $\varphi = 0$. In this case, $P_\nu = 0$, $q_0 = 0$, and taking into account the approximation of the modified Bessel functions for large values of the argument, in which we neglect all terms in the expansion other than the first, we have from (17) with $r > 0$

$$\tilde{T}(r, s) = \frac{t_0}{s}\frac{\varkappa_0^2}{\gamma^2} - \frac{t_0}{s}\left(\frac{\varkappa_0^2}{\gamma^2} - \frac{\chi_1}{\omega_1}\right)\sqrt{\frac{b_1}{r}}\exp\left[-\gamma\,|r - b_1|\right]. \tag{18}$$

Transforming in (18) to the original function and writing it in dimensionless form, we obtain

$$\begin{aligned}\vartheta = 1 - \exp(-\mathrm{Bi}_0\mathrm{Fo}) + \frac{1}{2}\sqrt{\frac{b_1^*}{\rho}}\Bigg\{\exp(-f_2^*\,\mathrm{Fo})\Bigg[\exp\left(\sqrt{f_1^*}\,|b_1^* - \rho|\right) \times \\ \times \operatorname{erfc}\left(\frac{|b_1^* - \rho|}{2\sqrt{\mathrm{Fo}}} + \sqrt{f_1^*\mathrm{Fo}}\right) + \exp\left(-\sqrt{f_1^*}\,|b_1^* - \rho|\right)\operatorname{erfc}\left(\frac{|b_1^* - \rho|}{2\sqrt{\mathrm{Fo}}} - \right. \\ \left. - \sqrt{f_1^*\,\mathrm{Fo}}\right)\Bigg] - 2\exp(-\mathrm{Bi}_0\mathrm{Fo})\operatorname{erfc}\left(\frac{|b_1^* - \rho|}{2\sqrt{\mathrm{Fo}}}\right)\Bigg\},\end{aligned} \tag{19}$$

where

$$\vartheta = \frac{T(\rho, \mathrm{Fo})}{t_0}; \quad f_1^* = \frac{\mathrm{Bi}_0 C_1^* - \mathrm{Bi}_1\lambda_1^*}{C_1^* - 1}; \quad f_2^* = \frac{\mathrm{Bi}_1\lambda_1^* - \mathrm{Bi}_0}{C_1^* - 1};$$

$$\mathrm{Bi}_k = \frac{\alpha_k\delta}{\lambda_k}, \quad k = 0, 1; \quad \mathrm{Fo} = \frac{a_0\tau}{\delta^2}; \quad b_1^* = \frac{b_1}{\delta}; \quad \rho = \frac{r}{\delta};$$

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-u^2)\,du.$$

In the stationary thermal regime, we have from (17)

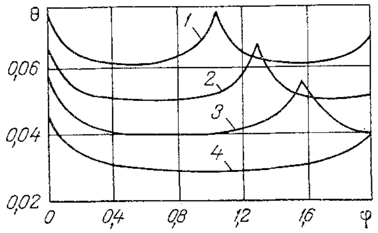

Fig. 4. Temperature profiles for different numbers of inclusions ($\rho = b_1^* = 5$): 1) n = 6; 2) 5; 3) 4; 4) 3.

$$\Theta = \frac{n}{2}\,\mathrm{Po}\,(R_1^*)^2\, D\left(F_0^* + 2\sum_{p=1}^{\infty} F_\nu^* \cos pn\varphi\right), \tag{20}$$

where

$$\Theta = \frac{T(\rho,\ \varphi) - t_0}{t_0};\quad \mathrm{Po} = \frac{q_0\delta^2}{\lambda_0 t_0};\quad R_1^* = \frac{R_1}{\delta_1};$$

$$F_\nu^* = K_\nu(\sqrt{\mathrm{Bi}_0}\ b_1^*)\, I_\nu(\sqrt{\mathrm{Bi}_0}\,\rho)\, S(b_1^* - \rho) + I_\nu(\sqrt{\mathrm{Bi}_0}\, b_1^*)\, K_\nu(\sqrt{\mathrm{Bi}_0}\,\rho)\times$$
$$\times S(\rho - b_1^*) - P_\nu^*\, I_\nu(\sqrt{\mathrm{Bi}_0}\, b_1^*)\, I_\nu(\sqrt{\mathrm{Bi}_0}\,\rho);$$

$$P_\nu^* = \frac{\nu K_\nu(\sqrt{\mathrm{Bi}_0}\, R^*) - \sqrt{\mathrm{Bi}_0}\, R^* K_{\nu+1}(\sqrt{\mathrm{Bi}_0}\, R^*)}{\nu I_\nu(\sqrt{\mathrm{Bi}_0}\, R^*) + \sqrt{\mathrm{Bi}_0}\, R^* I_{\nu+1}(\sqrt{\mathrm{Bi}_0}\, R^*)};\quad R^* = \frac{R}{\delta};$$

$$D = \left\{1 + \frac{n}{2}(R_1^*)^2\left[(\mathrm{Bi}_1\lambda_1^* - \mathrm{Bi}_0)\, D_1^* - (\lambda_1^* - 1)\frac{\sqrt{\mathrm{Bi}_0}}{b_1^*}\, D_2^*\right]\right\}^{-1};$$

$$D_1^* = K_0(\sqrt{\mathrm{Bi}_0}\, b_1^*)\, I_0(\sqrt{\mathrm{Bi}_0}\, b_1^*) - P_0^*\, I_0^2(\sqrt{\mathrm{Bi}_0}\, b_1^*) +$$
$$+ 2\sum_{p=1}^{\infty}\left[K_\nu(\sqrt{\mathrm{Bi}_0}\, b_1^*)\, I_\nu(\sqrt{\mathrm{Bi}_0}\, b_1^*) - P_\nu^*\, I_\nu^2(\sqrt{\mathrm{Bi}_0}\, b_1^*)\right];$$

$$D_2^* = H_0^* + 2\sum_{p=1}^{\infty} H_\nu^*;$$

$$H_\nu^* = -\frac{\nu}{\sqrt{\mathrm{Bi}_0}\, b_1^*}\, I_\nu(\sqrt{\mathrm{Bi}_0}\, b_1^*)\, K_\nu(\sqrt{\mathrm{Bi}_0}\, b_1^*) + \frac{1}{2}\left[K_\nu(\sqrt{\mathrm{Bi}_0}\, b_1^*)\times\right.$$
$$\left.\times I_{\nu+1}(\sqrt{\mathrm{Bi}_0}\, b_1^*) - K_{\nu+1}(\sqrt{\mathrm{Bi}_0}\, b_1^*)\, I_\nu(\sqrt{\mathrm{Bi}_0}\, b_1^*)\right] - P_\nu^*\, I_\nu(\sqrt{\mathrm{Bi}_0}\, b_1^*)\times$$
$$\times\left[\frac{\nu}{\sqrt{\mathrm{Bi}_0}\, b_1^*}\, I_\nu(\sqrt{\mathrm{Bi}_0}\, b_1^*) + I_{\nu+1}(\sqrt{\mathrm{Bi}_0}\, b_1^*)\right].$$

Using formulas (19) and (20), the temperature field in a glass plate ($\lambda_0 = 0.798$ W/m·K; $C_0 = 1642.2$ J/m^3·K) with metallic inclusions ($\lambda_1 = 15.12$ W/m·K; $C_1 = 3788.4$ J/m^3·K) was computed. In this case, it was assumed that $\mathrm{Bi}_0 = 0.01$; $\mathrm{Bi}_1 = 0.1$.

Figure 2 shows the results of a calculation of the temperature profiles for an infinite plate in the section $\varphi = 0$ for short times with $b_1^* = 5$. It is evident from the graph that the temperature increases with time.

The calculation for the stationary temperature regime was carried out for Po = 1.0; $R_1^* = 0.1$; $R^* = 10$. Figure 3 shows the temperature profiles as a function of the polar radius with $\varphi = 0$, n = 6 for different values of the radius of the circle on which the inclusions were placed. Figure 4 shows the temperature profiles as a function of the polar angle with $\rho = b_1^* = 5$ for different numbers of inclusions. As the radius of the circle on which the inclusions are placed increases, the temperature of the plate decreases. Increasing the number of inclusions with fixed radius of the circle on which they are placed increases the temperature.

NOTATION

T, temperature in the plate with the inclusions; r, polar radius; φ, polar angle; τ, time; $\lambda(r,\ \varphi)$, coefficient of thermal conductivity; $\alpha(r,\ \varphi)$, heat transfer coefficient; $C(r,\ \varphi)$, volume heat capacity; $W(r,\ \varphi,\ \tau)$, specific intensity of the heat sources; δ, half thickness of the plate; $\delta(x)$, Dirac's delta function; $\bar{T}$, finite Fourier cosine transform of the temperature; p, parameter for this transformation; $\tilde{T}$, Laplace transform of the temperature; s, its parameter; $I_\nu(x)$, the Bessel function with imaginary argument of order ν; $K_\nu(x)$, the MacDonald function of order ν; ϑ and Θ, dimensionless temperature; Po, Pomerantz number; Bi, Biot number; Fo, Fourier's number; ρ, dimensionless polar radius; b_1^*, dimensionless radius of the circle on which the inclusions are placed; R^*, dimensionless radius of the plate.

LITERATURE CITED

1. Yu. M. Kolyano and V. A. Volos, "Equations of thermoelasticity for nonuniform cylindrical plates," in: Physicomathematical Fields in Deformed Media [in Russian], Naukova Dumka, Kiev (1978), pp. 117-125.
2. V. Kech and P. Teodoresku, Introduction to the Theory of Generalized Functions with Applications in Engineering [in Russian], Mir, Moscow (1978).
3. G. Korn and T. Korn, Mathematical Handbook for Scientists and Engineers, 2nd edition, McGraw-Hill (1967).
4. I. F. Obraztsov and G. G. Onanov, Structural Mechanics of Skewed Thin-Walled Systems [in Russian], Mashinostroenie, Moscow (1973).

NUMERICAL METHOD FOR SOLVING HEAT-CONDUCTION PROBLEMS FOR TWO-DIMENSIONAL BODIES OF COMPLEX SHAPE

Yu. K. Malikov and V. G. Lisienko

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A finite-difference scheme is described for a curvilinear orthogonal net which permits the use of a single algorithm for calculating bodies of various shapes.

The construction of curvilinear difference nets by calculating the conformal mapping of a canonical region (rectangle) into the given region was described in [1]. Unlike a rectangular net which is typical for the finite-difference method [2-4] and in practically important cases bears little resemblance to the boundaries of the body, an orthogonal net reflects the nature of the boundary, and has no nonregular nodes. An orthogonal net is more convenient to work with than the nets commonly used in the method of finite elements [5] since all quantities referring to the nodes of such a net (e.g., their coordinates) can be written in the form of a rectangular matrix. Using equations of the elliptic type as an example, variational-difference schemes for such nets were discussed in [6].

An analysis in [7] showed that finite-difference schemes have distinct advantages over variational-difference schemes in solving heat-conduction problems. For this reason the finite-difference method is of great interest for solving heat-conduction problems with orthogonal nets [8]. The practical use of the algorithm obtained confirmed its adequate accuracy, high speed, and, what is particularly important, the simplicity of its application for bodies of various shapes. However, it is not clear from [8] under what conditions and at what rate the scheme converges to the solution of the original equation.

We describe a procedure which employs a set of standard programs to automate the process of solving the heat-conduction equation for a broad class of two-dimensional regions. If an orthogonal net is constructed by conformal mapping, the rate of convergence of the finite-difference scheme can be estimated.

Let the function $F(w) = F(u + iv)$ map the rectangle G conformally into the region G^* in the (x, y) plane. We assume that $F(u + iv)$ is known and that $\partial F/\partial w$ exists and is finite on the boundary Γ of the rectangle. The latter implies that G^* is a curvilinear quadrangle in which all the angles are right angles.

We consider the problem for the heat-conduction equation posed in G^*:

$$c\frac{\partial T}{\partial \tau} - LT = Q, \tag{1}$$

S. M. Kirov Ural Polytechnic Institute, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 40, No. 3, pp. 503-509, March, 1981. Original article submitted February 29, 1980.

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